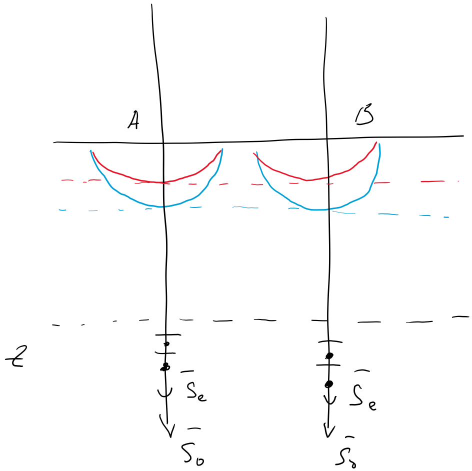
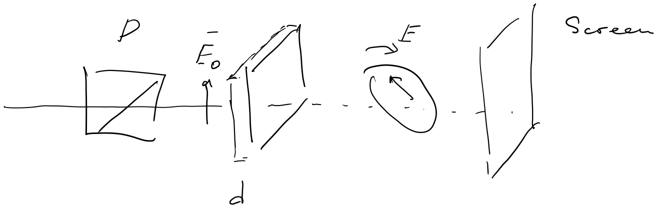
Interference of polarized waves


A
B
z
$\bar{S}_e$
$\bar{S}_o$
$\bar{S}_e$
$\bar{S}_o$

Since ordinary and extraordinary waves penetrate with different speed, the will be phase difference.

Let's build simple setup:


P
$\bar{E}_0$
$\bar{E}$
Screen
d

Natural light comes to polarizer P and then goes through single axis crystal plate of thickness d that is cut along optical axis z.

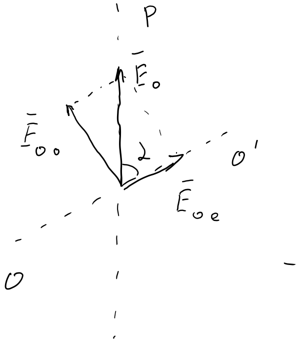
Optical path difference

$$\Delta = d(n_o - n_e)$$

Since there is path difference, there will be phase difference.

$$\Delta\varphi = \frac{2\pi}{\lambda}\Delta = \frac{2\pi}{\lambda} d(n_o - n_e)$$

Wave amplitudes can be derived the following:

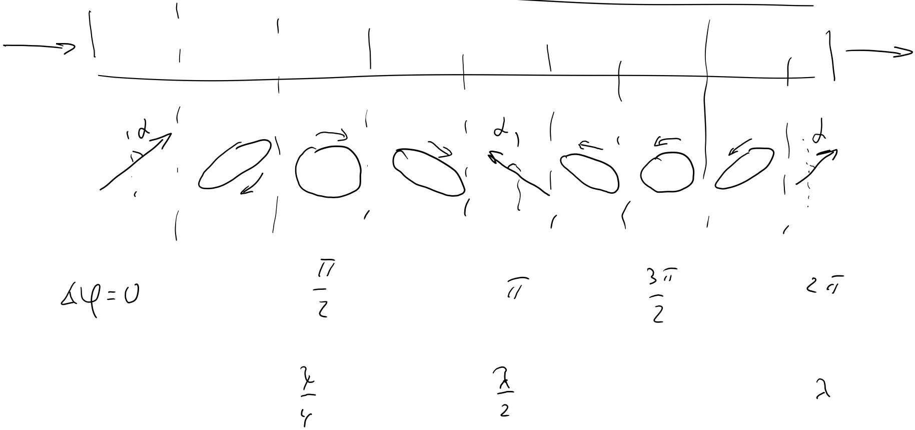

P
$\bar{E}_0$
$\bar{E}_{oo}$
α
O'
$\bar{E}_{oe}$
O

$$\begin{cases} E_{oe} = E_0 \cos\alpha \\ E_{oo} = E_0 \sin\alpha \end{cases}$$

If $\alpha = \frac{\pi}{4}$, then $E_{oe} = E_{oo}$

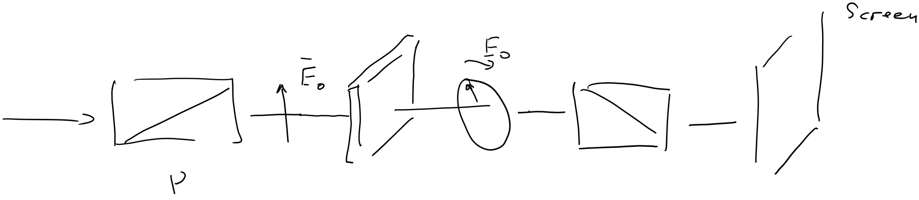
Though we will not observe interference in terms we discussed it, we know that if we sum up two ortogonal polarized coherent waves, we will have elliptically polarized light.

$$\begin{cases} E_{oe} = E_0 \cos\alpha \\ E_{oo} = E_0 \sin\alpha \end{cases} \quad \Delta\varphi = \frac{2\pi}{\lambda}\Delta = \frac{2\pi}{\lambda} d(n_o - n_e)$$

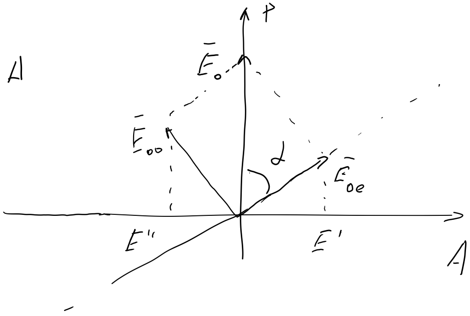

α
α
α
$\Delta\varphi = 0$
$\frac{\pi}{2}$
π
$\frac{3\pi}{2}$
2π
$\frac{\lambda}{4}$
$\frac{\lambda}{2}$
λ

Since the phase difference is constantly changing, the polarization state evolve from linear to circular and back to linear.

Cutting different thicknesses we can have $\frac{\lambda}{4}$ or $\frac{\lambda}{2}$ waveplates.


Screen
$\bar{E}_0$
$\vec{E}_0$
α
P

① $P \perp A$


P
$\bar{E}_0$
$\bar{E}_{00}$
α
$\bar{E}_{0e}$
E''
E'
A

$$\begin{cases} E_{0e} = E_0 \cos\alpha \\ E_{00} = E_0 \sin\alpha \end{cases} \Rightarrow \begin{cases} E' = E_{0e} \sin\alpha = E_0 \sin\alpha \cos\alpha \\ E'' = E_{00} \cos\alpha = E_0 \sin\alpha \cos\alpha \end{cases}$$

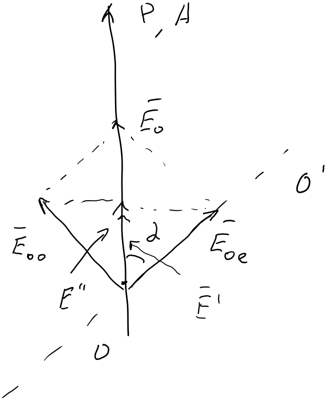
$$I_\perp = I' + I'' + 2\sqrt{I' I''} \cos(\Delta\varphi + \pi)$$

$$I_\perp = 2 I_0 \sin^2\alpha \cos^2\alpha + 2 I_0 \sin^2\alpha \cos^2\alpha \cos(\Delta\varphi + \pi)$$

$$I_\perp = I_0 \sin^2 2\alpha \sin^2 \frac{\Delta\varphi}{2}$$

$$\Delta\varphi = \frac{2\pi}{\lambda} d (n_0 - n_e)$$

② $P \parallel A$


P, A
$\bar{E}_0$
O'
$\bar{E}_{oo}$
$\bar{E}_{oe}$
α
E''
$\bar{E}'$
O

$$\begin{cases} E_{oe} = E_0 \cos\alpha \\ E_{oo} = E_0 \sin\alpha \end{cases}$$

$$\Downarrow$$

$$\begin{cases} E' = E_{oe}\cos\alpha = E_0 \cos^2\alpha \\ E'' = E_{oo}\sin\alpha = E_0 \sin^2\alpha \end{cases}$$

$$I_{\parallel} = I_0 \left(1 - \sin^2 2\alpha \sin^2 \frac{\Delta\varphi}{2}\right)$$

For $\alpha = \frac{\pi}{4}$

$$\begin{cases} I_{\perp} = I_0 \sin^2 \frac{\Delta\varphi}{2} \\ I_{\parallel} = I_0 \left(1 - \sin^2 \frac{\Delta\varphi}{2}\right) \end{cases}$$

Let's assume $\Delta\varphi = \frac{2\pi}{\lambda} d(n_o - n_e) = \begin{cases} 2m\pi, & \lambda_1 \\ (2m+1)\pi, & \lambda_2 \end{cases}$ 🔴 🟢

Hence,

$$\sin^2 \frac{\Delta\varphi}{2} = \begin{cases} 0, & \lambda_1 \\ 1, & \lambda_2 \end{cases}$$

$$\begin{cases} I_{\perp}(\lambda_1) = 0 \\ I_{\perp}(\lambda_2) = I_0 \end{cases}$$ 🟢

$$\begin{cases} I_{\parallel}(\lambda_1) = I_0 \\ I_{\parallel}(\lambda_2) = 0 \end{cases}$$ 🔴

Demonstration
